Construction and Diversification of a Portfolio That Replicates the BBB Credit Index

By

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In this paper we are going to examine the practical problem of constructing a BBB rated portfolio by utilizing a subset of the BBB universe issues and issuers. The purpose of this portfolio is to track the returns of the BBB index within a given range of tracking error and a given confidence level.

The challenge in structuring a portfolio that replicates a fixed income index or a subset of the index arises from the fact that one cannot invest in all the bonds in the index. A replicating portfolio will utilize a subset of the bonds that the index consists of and will have the objective of generating returns within a band of no more than x basis points around the returns of the index.

Tracking the returns of a fixed income portfolio is an easier task than for other asset classes as there are well defined systematic risk factors that can be matched to those of the index and thus create a common exposure to them. For example, matching the key rate durations of the replicating portfolio ("RP") to those of the BBB benchmark ("B") will lead to a RP that will be expected to perform in line with B when the general level of interest rates changes as the yield curve reshapes. Similarly, if the RP has the same key rate spread durations to the B then both portfolios will be affected in the same manner from a general market widening or tightening or BBB spreads. A RP yield that matches the B yield will ensure that both portfolios will have the same income generating ability.

Assuming all the systematic risk factors have been matched the main risk that could cause the two portfolios to have significantly different performance profiles arises from the non-systematic risk of the two portfolios. Each individual bond has a diversifiable risk that, in theory, the market does not compensate for. The main risk associated with a specific bond arises from the likelihood that a given bond will get downgraded due company specific issues and its spread will widen, while the rest of the market will not follow this widening. There are about 1,620 bonds in the BBB index. If we treat these bonds as the "market portfolio" in the sense of the Capital Asset Pricing Model, the fact that some small number of these bonds related to a specific issuer might get downgraded and thus underperform the other bonds in B, and the fact that these bonds might be held in different amounts in B then the RP becomes the source of the return discrepancy between the two portfolios. If we held each bond in the RP with the same percentage as the market portfolio, then the security specific risk of downgrades has been obviously diversified and the returns of the two portfolios should be identical. The risk that we accept in a RP, a risk that the market will not compensate us for, is that since we utilize a small subset of the B to replicate it, and thus the percents allocated to each bond will not match to those in B. The remainder of this study will address the tracking error that is introduced in a RP by the finiteness of the bonds used and its relation to the number of bonds used.

Let us start by defining the Tracking Error ("TE") of a replicating portfolio as the difference between the returns of the B and the RP. We assume that BBB returns and the RP returns follow a Geometric Brownian motion with the same drift (equal to the matched yield of the two portfolios), a common random component arising from the common sources of systematic risk, (we assume that the systematic risk factors have been matched) and a non- common random component (s dZ), where dZ is a white noise and s the volatility of the BBB returns arising from the issue specific randomness of returns. If there are N bonds in the market portfolio, and we assume zero correlation of non-systematic risks, and each has a volatility σ from issue specific events, then, $s^2 = \sigma^2 / N$. When N is large enough, then s will be quite small - the risk is diversifiable. We only focus on the issue specific risk since the systematic risk contribution to returns will be common to both the RP and the B and so they will cancel out when the returns are subtracted to obtain the TE. For the RP the issue specific volatility is, σ^2 / n, where n the number of issues used in the RP.

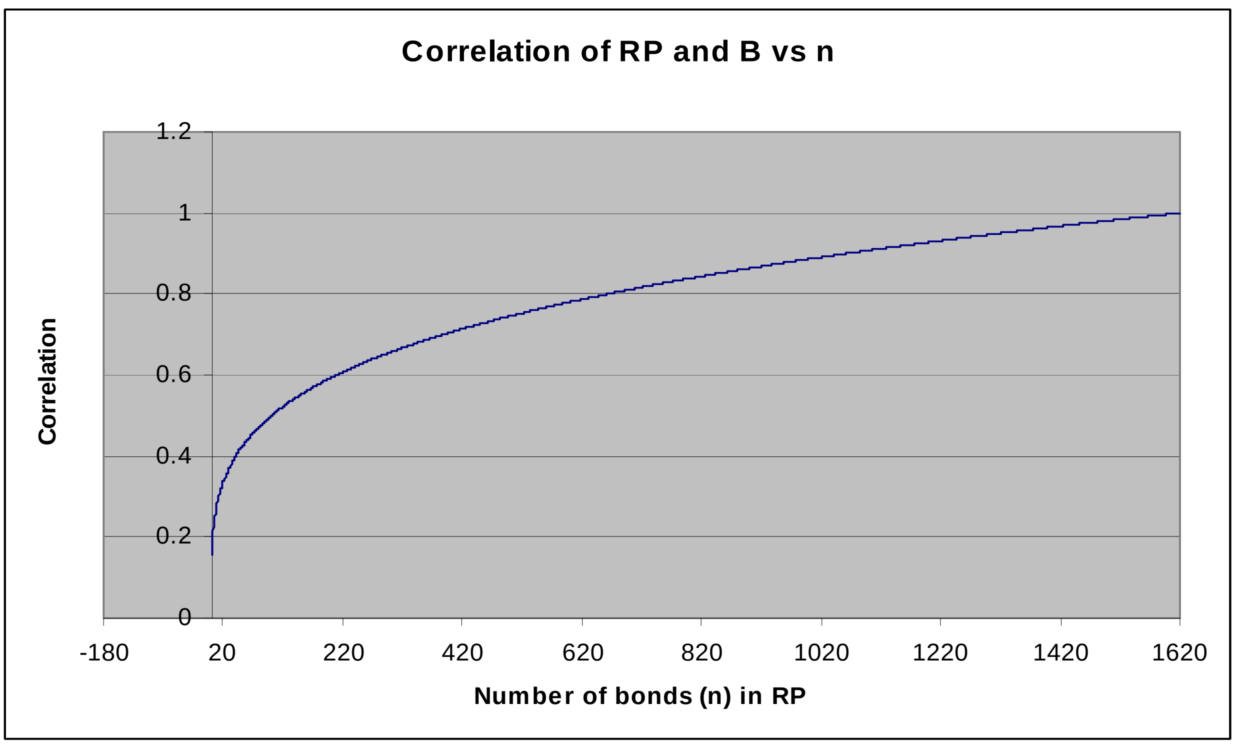
So, the TE is a random variable ($R_{TE} \equiv R_{RP} - R_B$), with a mean of zero (since the RP is expected to follow the returns of B as the systematic risks are matched) and a variance, Var(TE). Obviously, if n=N the Var(TE)=0 and we have perfect replication. For n<N the variance starts to increase, and obviously at the other limit when n=1 the variance is just, σ^2, that of the issue specific volatility.

Next we will model the Var(TE) as a function of n, with N=1,620 and try to get some insight as to the dependence of the TE on the quantity of the bonds in the RP. The Var(TE) can easily be obtained from the individual variances of the RP and B. The one piece that we need to consider is the correlation ρ between the returns of the RP and B that are attributed to issue specific events.

The value of ρ should be between 0 and 1. It should depend on n, since if n=1 then ρ should be 0, as a one bond RP should not have any correlation with the market portfolio (remember that here we are just focusing on the issue specific component of returns which is what remains when we subtract the returns of B from the returns of the RP to obtain the TE). For n=N, ρ=1 as the market portfolio perfectly correlates with itself. In between, we are looking for a function that increases rapidly with n at first and then levels off for larger values of n. Such a function is:

$$\rho = \left(\frac{n}{N}\right)^{\alpha},$$

with 0<α<1. α=1 is not very realistic as the incremental change in ρ should not remain the same as we add more and more bond to the RP. The selection α=(1/4) seems to offer a realistic choice and is shown below. As we will see later, the TE is not very sensitive to the choice of α.

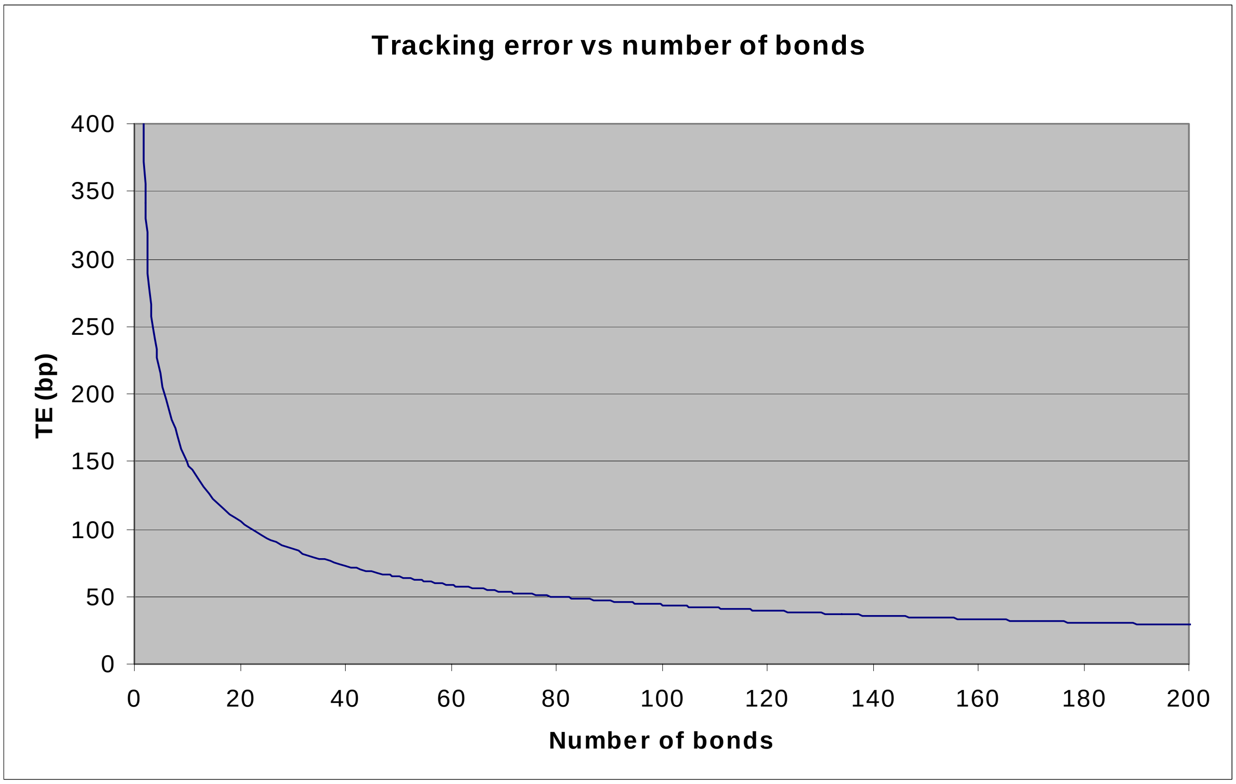


Utilizing this choice and the well known result of adding variances, we obtain the variance of the TE as:

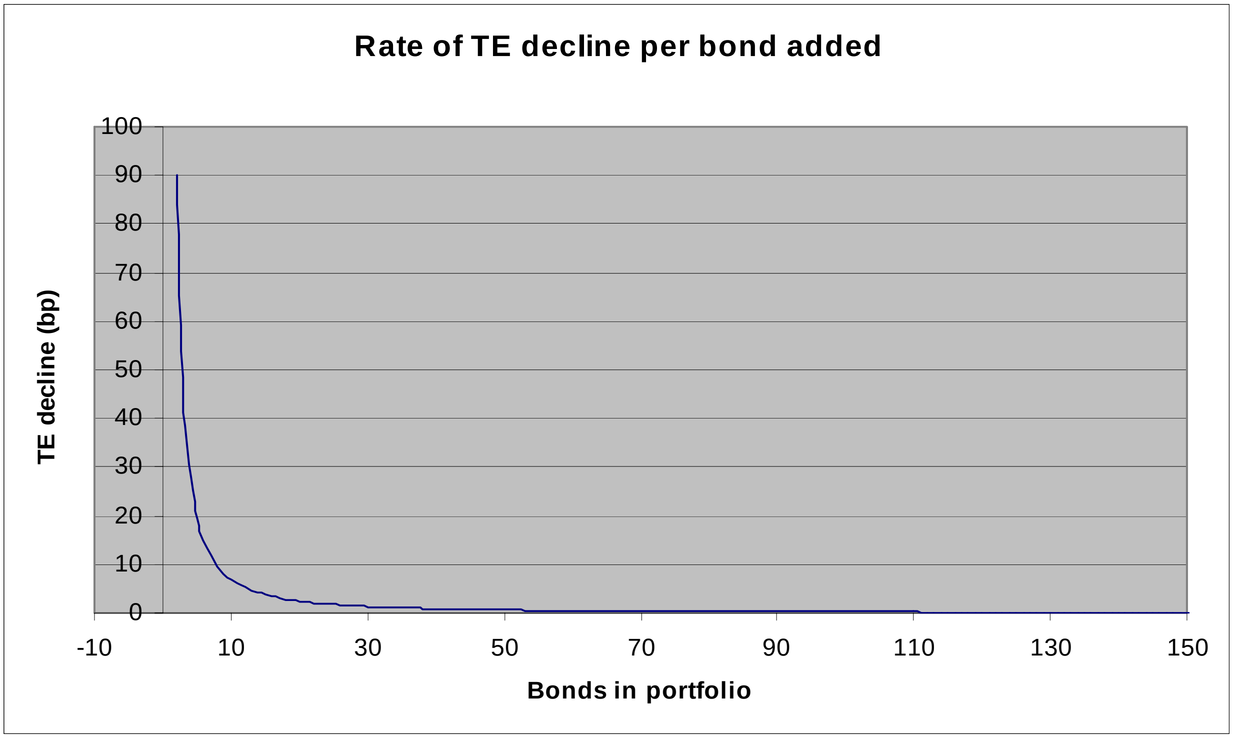
$$\sigma_{TE}^2 = \frac{\sigma^2}{N} + \frac{\sigma^2}{n} - 2\left(\frac{n}{N}\right)^{1/4} \frac{\sigma^2}{\sqrt{nN}}$$

This is the formula that we are going to utilize to explore the dependence of the TE on n. The assumption here is that B and the RP consist of N and n equally weighted bonds respectively. The second half of this study will analyze the TE for portfolios of bonds with unequal weights.

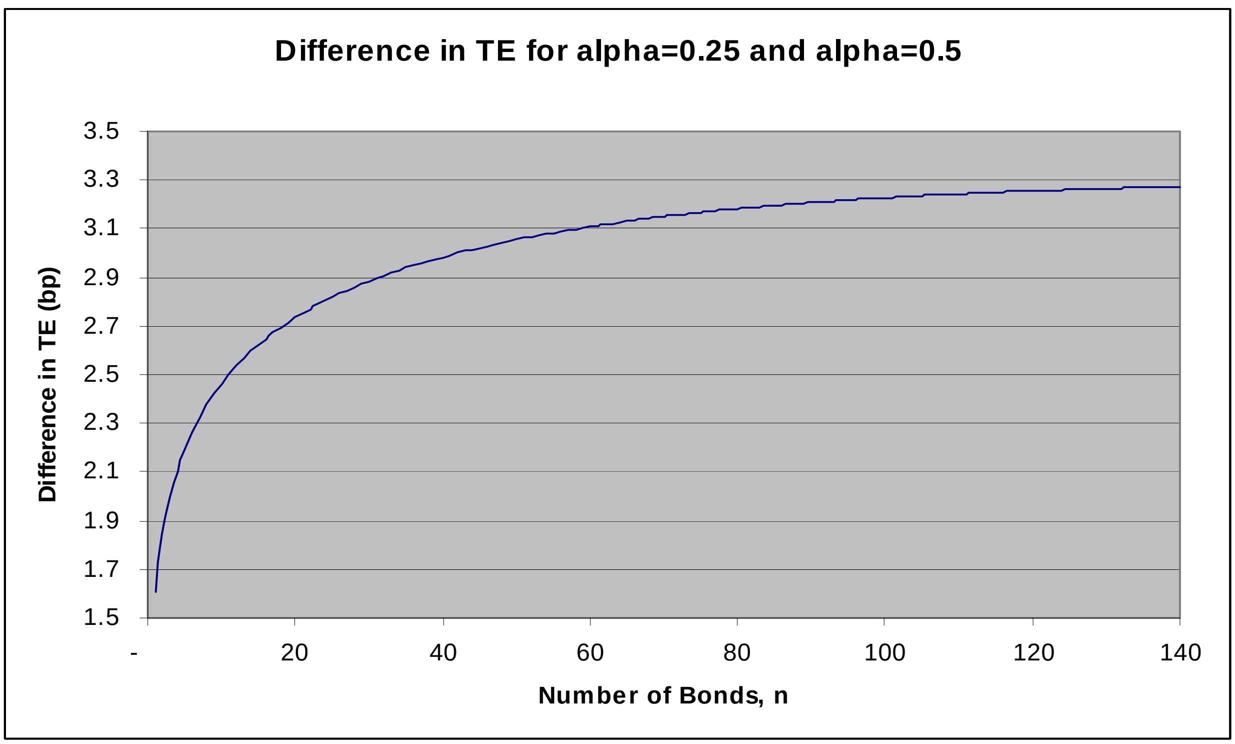
The chart below shows the TE as a function of n, when the diversifiable volatility of a BBB issue is 486 basis points, as Dynkin, Hyman and Konstantinovsky of Lehman Brothers[1] have calculated.



We see that increasing the number of bonds used in a BBB RP, dramatically reduces the TE, but after a point the impact of adding more bonds is diluted as we can see from the chart below.



The chart below shows the difference in the TE when α=0.25 and α=0.5. We see that the difference is less than 3.5 basis points and shows that the choice of α is not critical for our analysis.



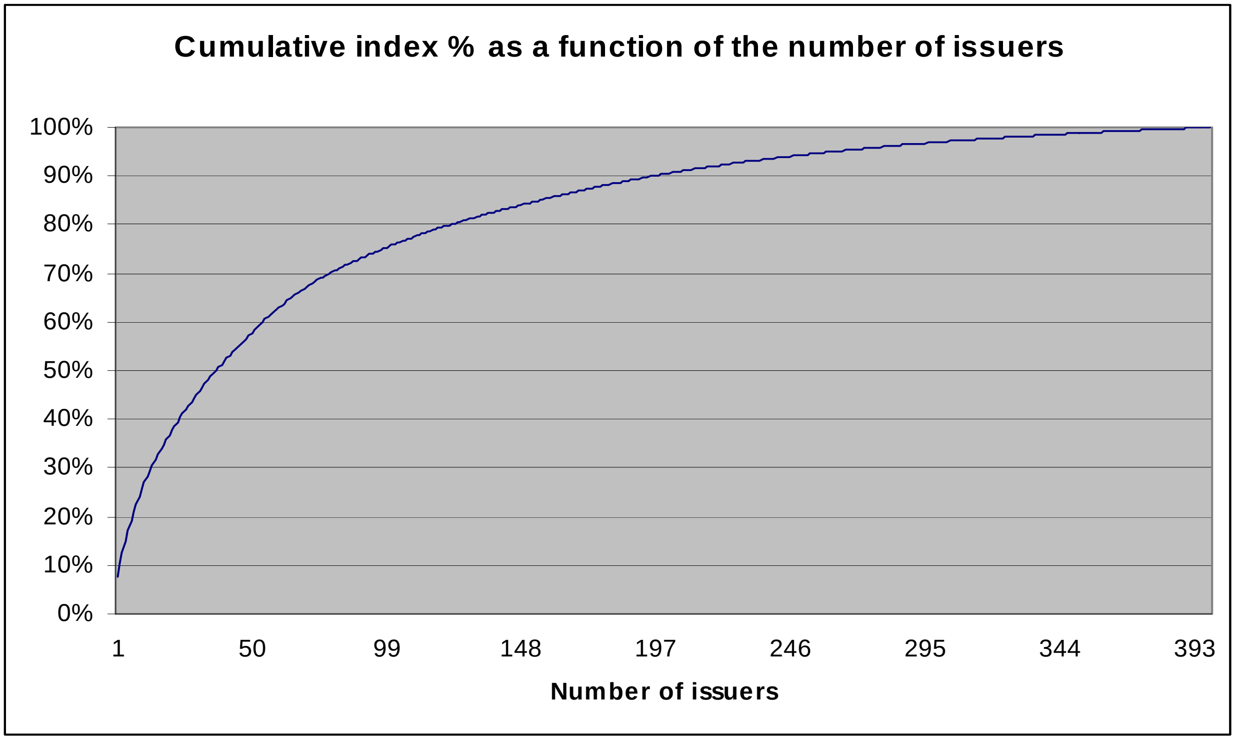
Utilizing the analysis above we see that it would take in excess of 80 equally weighted bonds (picked at random from the universe of 1620 bonds) in the RP to bring the TE down to less than 50 basis points, annualized.

Market Structure Analysis

The assumptions that went into the discussion above work quite well with indices composed of a large number of issues. When replicating a subset of a big market index, such as the BBB index that has a small number of issuers, then since the issuer representation in the BBB index is not uniform, one can reduce the TE by matching a number of the largest issuers and replicating the rest. Utilizing the market structure of the BBB index, we see that the top 10 issuers account for almost 25% of the index. There are 400 issuers in the BBB universe. Each of these issuers represents a different percent of the B and all the issues of a specific issuer are perfectly correlated with each other as far as the non-systematic diversifiable risk is concerned. We ignore second order effects such as the fact that the credit spread curve might steepen when a downgrade occurs, or that it might invert in cases of liquidity crisis and bankruptcy fears.

The actual form of the diversifiable risk for RP purposes is the deviation in the % allocation in the RP from that of the B. This is where the issuer specific risk arises. If we matched the % of the different issuers exactly with those of the index there would be no tracking error. The TE arises from the allocation of different percentages to various issuers than those in the B.

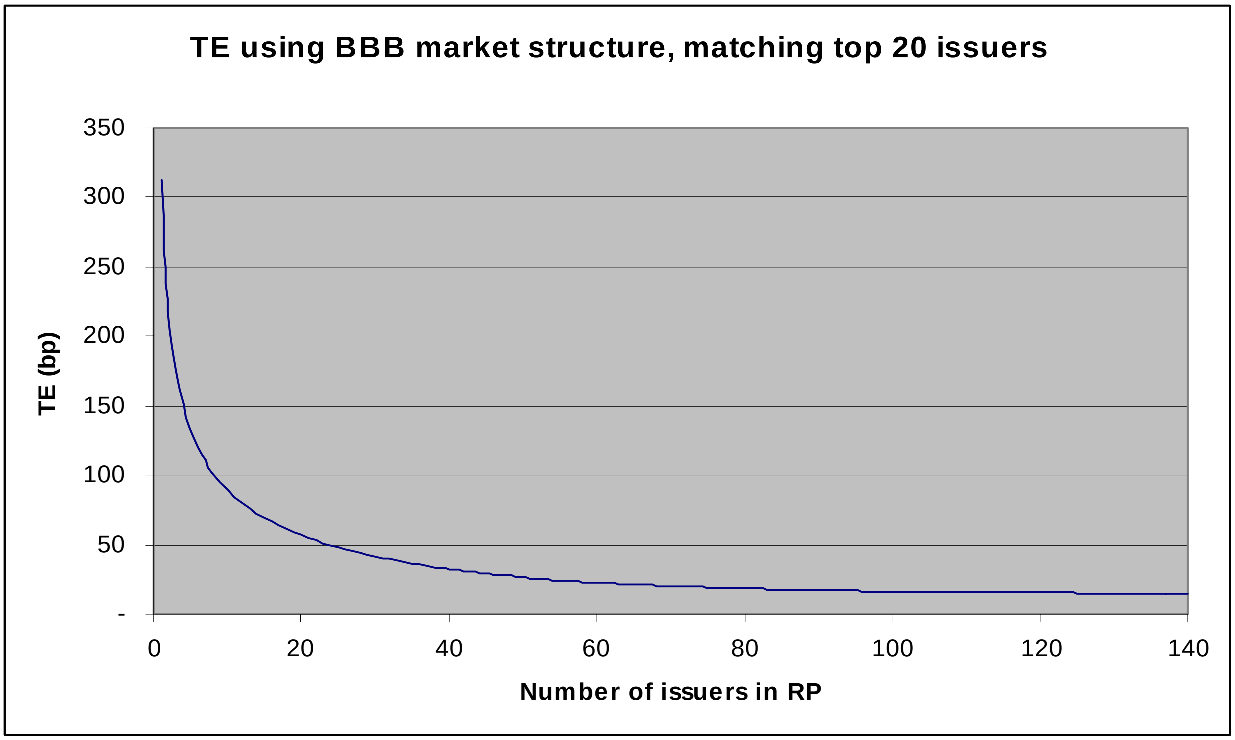
The following shows the cumulative % exposure of the index as a function of the number of issuers. The top 50 issuers account for about 58% of the market value of the index instead of the 12.5% the equal weight model assumes.



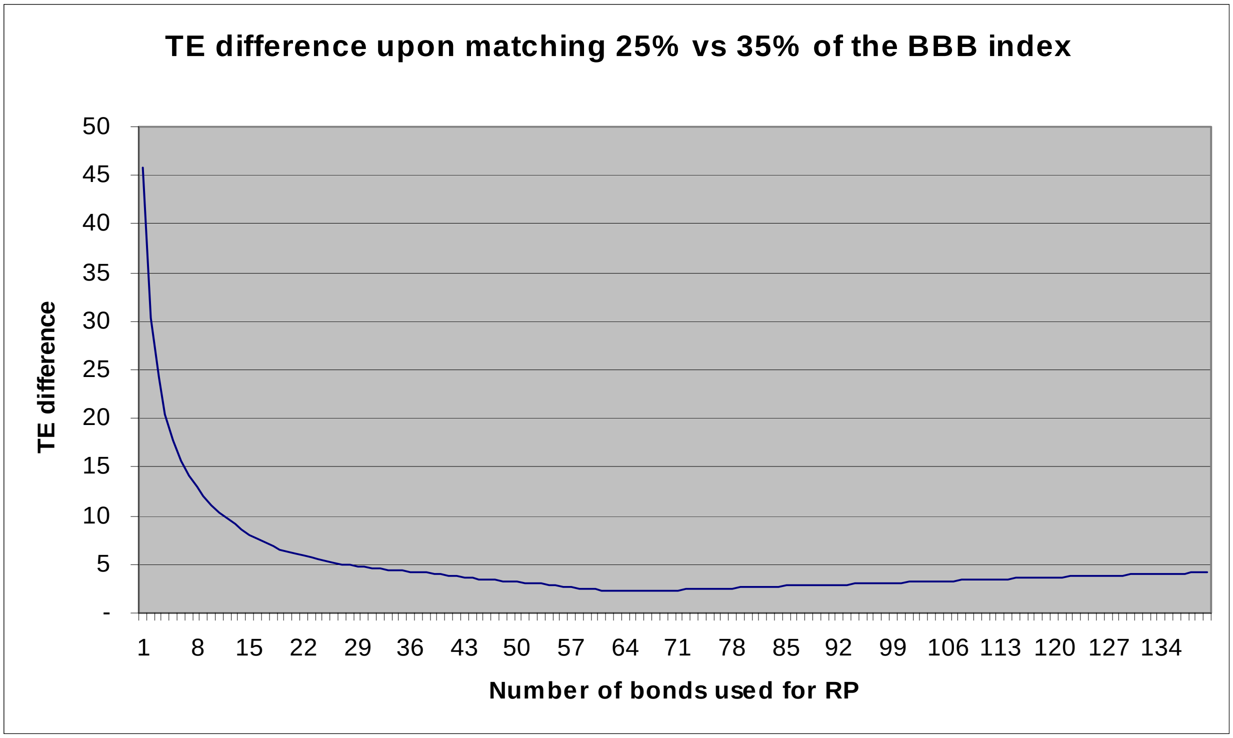
This fact allows us to match the top index issuers and replicate the remainder of the issuers and achieve a lower TE.

For the following analysis we assume that each issuer's diversifiable risk is uncorrelated with that of other issuers. We match the top 20 issuers, or 35% of the BBB index in terms of percent exposure and other systematic risk factors. For the remainder 65% of the BBB index we replicate by allocating equally amongst the next n issuers in size, each one with a percent 0.65/n. We could attempt to model the function $x(i)$ for i=1…N, where N here represents the number of issuers, 400, as a monotonically declining function or utilize the actual % values. We proceed with the second option as the differences will be minimal.

Since all issues of an issuer are assumed perfectly correlated, the volatility of the issuer will be just that of the issue, σ. The numerical value of 486 basis points will be used. The contribution of issuer i to the overall index non-systematic risk is: $x(i)\sigma^2$. Assume we match the first M issuers, in this case M=20. There is no TE for this part of the portfolio arising from issuer specific events. For the remainder N-M issuers, we replicate by allocating uniformly amongst n issuers, ($N - M \leq n \leq N$). We are going to analyze the dependence of TE to n. The TE here arises from the fact that the percent allocated to each of the n bonds in our RP is 0.65/n, instead of $x(i)$ for B. For the first n issuers after the M matched, the percent difference will be $\left(x(i) - \frac{0.65}{n}\right)$, and for the remainder issuers nor represented in the RP, just $x(i)$. These will be the allocations for the issuers in a hypothetical portfolio generating the tracking error. The assumption of zero correlation allows us to just add the variances and get the result shown below.



Obviously, if one matches more issuers the TE declines. The amount of decline when one goes from matching the top 10 to matching the top 20 issuers is shown below.



If one uses a high enough number of issuers in the RP, greater than 30, the TE difference between the two portfolios is less than 5 basis points.

Balancing the liquidity and availability of certain issuers against the objective of keeping the tracking error as low as possible, it appears that for the BBB index, matching 35% of the index, or 20 issuers, and then utilizing the next 40 issuers for the RP should result in a tracking error of 32 basis points between the returns of the BBB index and those of the matching+replicating portfolio on an annualized basis.

Assuming that the TE is a random variable with a mean of zero and a standard deviation of 32 basis points, we can establish confidence level intervals that would allow us to place desired confidence level that the TE would be less than a given number, x% of the time. The 1 standard deviation interval means that 67% of the time the matching+replicating portfolio returns will be within + or – 32 basis points to those of the BBB index. Going to higher confidence levels means that the band widens. For example, if we want the confidence level to be 95% then the band becomes 2 times 32 or 64 basis points, and so forth.